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A Quantitative Method for the Determination of Hardness in Wheat.

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By H. F. Roberts.

SUMMARY.

1. The present bulletin gives an account of an apparatus designed to determine quantitatively, in terms of the mean crushing-point of the kernel, different degrees of hardness in wheat; this factor, or rather combination of factors, being considered indicative of superior milling quality in wheat of the *Triticum vulgare* types.

2. The ulterior purpose of this investigation is to establish a laboratory method for determining the relative hardness of pure strains of wheat; and finally to ascertain the degree of correlation existing between the hardness of the kernel and the chemical and physical characters of the gluten.

3. The present investigation involves the determination of the necessary number of kernels of wheat which must be used in order to arrive at a sufficiently correct average mean crushing-point for any given pure strain or variety.

4. It has been ascertained by a thorough investigation of two pure strains of wheat, the one hard, the other soft, that a correct average or mean crushing-point, accurately expressing in grams the degree of hardness of the particular races under investigation, was reached by taking the mean of the crushing-points of 350 kernels. Manifestly, quantitative estimates of hardness in any given pure strain of wheat will vary somewhat with the season and the locality.

5. The perfect concordance of the results of the experiments with the two strains of wheat chosen, Nos. 1115 and 1112, which have mean crushing-points of 6817 grams and 11,802 grams, respectively, for a total of 2700 kernels each, indicates that the facts disclosed by the investigations are probably general.

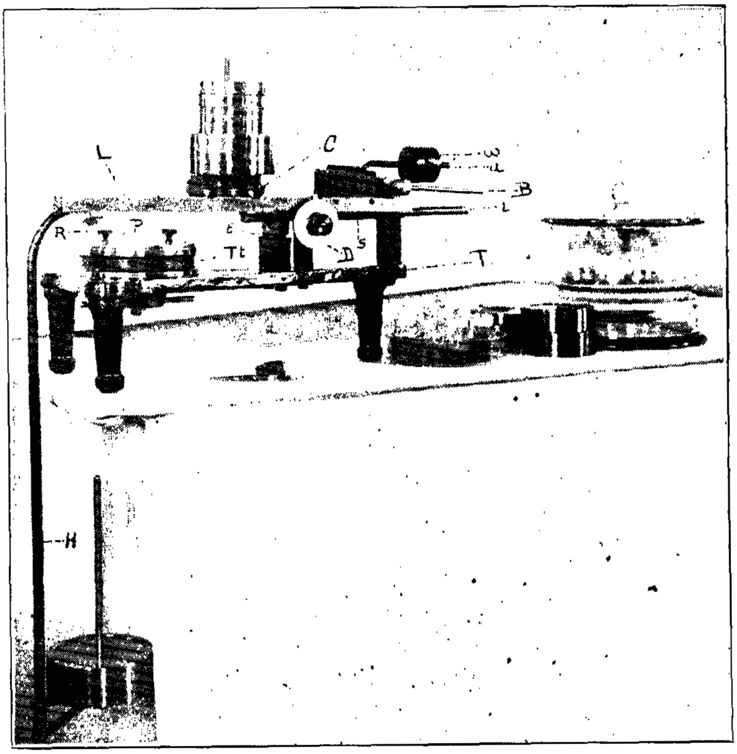
METHODS OF DETERMINATION.

To practical men of long experience in buying wheat of milling grades in the market, the present rough system of grading wheat is no doubt easy of exact application to particular cases.

In our work in wheat breeding, however, at the Kansas State Agricultural College and Experiment Station, it became necessary for scientific purposes to develop an exact and quantitative method for determining degrees of hardness in the wheat grain. It was found that no existing type of apparatus would in any way answer the purpose, and that it was therefore necessary to devise and construct one especially intended for this work. After some preliminary experiments, an instrument was designed with the aid of Mr. Wm. Gaertner, of Chicago, to whose ingenious and skillful assistance we are much indebted. This grain crusher, as illustrated in plate I, has been in constant use in our wheat investigations for the past two years, and has proved entirely successful and satisfactory in its operations. The new model of the machine, which embodies some needed improvements, is now to be had of Wm. Gaertner & Co.

The principle upon which the instrument depends for its

PLATE I.



construction is that of a lever in which the forces are located on the same side of the fulcrum. A table *T* supports a lever arm *L*, balanced on a knife-edge at *e* by a counter weight, *w,* which is adjustable along an arm, *a.* A carrier *C* which is removable runs on the top of the lever arm as a track upon four flanged wheels. A sliding rack arm *r,* running in a sleeve *s,* and actuated by a pinion set in motion by the revolution of the dial *D,* forms a connection beneath with the carrier *C.* The revolution of the dial *D* by the hand slides the carrier thus out along the track of the lever arm *L,* which bears a scale graduated to tenths and hundredths. At the end of the lever arm is a hanger *H,* which rests upon it with a knife-edge bearing surface. At the base of this hanger is a plate with an upright pin, on which perforated weights are strung. At *R,* fastened to the under side of the hanger end of the lever arm, is a ram or hammer which crushes the grains in succession, the latter lying in the pans *P* of the turntable or capstan *Tt,* which is revolved on a notched gear by means of spokes underneath the turntable (in the first model above it).

The weights are of brass, calibrated by the United States Bureau of Standards, and are in 1, 2, and 5 kilogram pieces.

The grains to be tested are first dried for seven days (in our our practice), at the boiling point of water, in the usual weighing bottles employed in analytical work, and which are then kept in a desiccator while the work is in progress. The grains are laid one at a time on the pans of the turntable, care being taken to place each grain in the same position (with the groove or crease down). Weights (preferably to nine kilograms for hard wheats) are mounted on the carrier *C,* which itself, detached, weighs a kilogram. The lever arm *L,* which is balanced before being weighted, is prevented from descending with the pull of the weighted hanger by means of a short lever at *B,* which operates on an eccentric brake. In operation the brake is slowly released after the machine has been weighted and the grain to be crushed has been brought into position. The ram *R* is thus carefully and gently brought to rest upon the upper surface of the kernel. The weighted carrier, which is at *O* on the lever arm scale, is now slowly moved out along its track by means of the rack-and-pinion gear mentioned, set in motion by revolving the disc *D.* By this means a constantly augmenting increment of weight is brought to bear upon the kernel, which finally gives way suddenly when the strain becomes too great. The crushing-point thus ascertained is the sum of the weight on the hanger arm, which, as it is brought to bear immediately over the kernel, is read directly, plus the fraction in 10ths and 100ths of the carrier weight, as read off on the lever arm scale. Thus, if the carrier weight (as we use it) amounts, with the carrier itself included, to 10 kilograms, and the grain when crushed finds the carrier pointer *P* at 7.2 on the lever arm, the pressure or weight which the ram *R* exerts on the kernel due to the influence of the carrier weight is .072 of 10 kilograms at that point, or 7.12 kilograms. By loading to 10 kilograms therefore the carrier weight can be read directly on the lever scale. With an odd number, such as 5 or 7 kilograms weight, the process in the case given of determining $\frac{5}{.072}$ would be tedious and time consuming. We therefore in our work with wheat, for which a combined carrier and hanger load of 15 kilograms is usually sufficient, load the carrier constantly to 10 kilograms, and increase the weight further if necessary before moving the carrier out. The dial *D* is graduated also to 10ths and 100ths of its circumference, in such manner that 10 spaces on the dial in revolution move the carrier pointer over one space on the lever arm scale. The dial thus reads in its smallest subdivisions to grams, in terms of the increasing pressure of the carrier weight due to its revolution.

The machine above described and figured is now in its second model, and has been thoroughly tested in our work of grading pure-bred wheats on a scale of hardness. In general, we find that "soft" wheats crush under a pressure of 6000 grams or less (13 pounds), "semi-hard" wheats at about 9000 grams (20 pounds), and "hard" wheats at 12,000 grams and over (26 pounds and over).

It immediately became a question, after a series of preliminary tests, as to how many kernels it might be necessary to crush in order to arrive at an approximately correct average estimate of the hardness of the race of wheat under experiment. To thls end two pure strains were selected—one, No. 1115, a soft wheat, and another, No. 1112, a hard strain.

From each of these races 2700 kernels were taken at random, and, after having been dried for seven days at the boiling point of water, were crushed in succession; the crushing-point in grams for each kernel being recorded separately. With the data thus obtained a series of groups was formed by taking *seriatim,* and consequently without selection, the data of the crushing-points as variates, thrown into nine successive groups, containing in each, respectively, 100, 150, 200, 250, and so on up to 500 variates. In this manner no single variate was used twice, and the means, standard deviations and errors of the means were calculated for each of the nine groups in the case of each of the two strains under experiment.

From the above data it was anticipated that a plotting of the errors of the mean crushing-points for successively increasing groups of variates might throw light on the minimum number of variates necessary to be crushed in order to arrive at a sufficiently correct mean crushing-point for any race of wheat.

In the case of No. 1115, for which the calculations were first made, the deviations from the mean were first taken as originally determined in grams. In the case of so many groups with such a large number of variates in most of them, and with all of the data expressed in thousands of grams, the reduction of the data became excessively tedious. However, the standard deviations and errors of the mean were calculated for the entire nine groups of No. 1115, using the data of the crushing-points exactly as found in grams. For comparison,

TABLE I.

Wheat No. 1115. Constants for 2700 kernels.

Groups of variates.	No. of variates in group.	A.—Crushing-points of variates in grams.				B.—Crushing-points of variates in kilos.			
		Mean crushing-point (gms.)	Error of mean crushing-point (gms.)	Standard deviation (grams).	Error of standard deviation (gms.)	Mean crushing-point (kilos).	Error of mean crushing-point (kilos).	Standard deviation (kilos).	Error of standard deviation (kilos).
Group I	100	7035	72.12	1069.32	51.00	7.00	0.0729	1.0802	0.0515
" II	150	7062	63.71	1156 85	45.05	7.10	0.0638	1.1587	0.0451
" III	200	6981	51.34	1076.55	36.31	7.00	0.0532	1.1155	0.0376
" IV	250	6943	56.93	1334.55	40.56	6.90	0.0571	1.3378	0.0404
" V	300	6852	45.67	1172.91	32.30	6.80	0.0462	1.1873	0.0327
" VI	350	6658	41.43	1149.15	29.30	6.60	0.0377	1.1558	0.0295
" VII	400	6560	42.90	1272.10	30.34	6.50	0.0434	1.2881	0.0307
" VIII	450	6499	35.35	1111.86	25.00	6.50	0.0351	1.1059	0.0282
" IX	500	6764	38.37	1271.89	27.13	6.71	0.0367	1.2159	0.0259

NOTE.—Multiplying the data under "B" by 1000, the close correspondence of the constants calculated from the crushing-points expressed in kilos and tenths of kilos, with those calculated from the data as originally determined in grams, will be readily seen.

TABLE II.

Wheat No. 1112. Constants for 2700 kernels.

Groups of variates.	No. of variates in groups....	Crushing-points of variates (reduced to kilos and tenths of kilos).			
		Mean crushing-point (kilos).	Error of mean crushing-point (kilos).	Standard deviation (kilos).	Error of standard deviation (kilos).
Group I...	100	12 3	0.1398	2 0729	0.0989
" II...	150	11.7	0.1234	2 2403	0.0872
" III...	200	11.7	0.1107	2.3215	0.0774
" IV...	250	11.5	0.1078	2 5259	0.0762
" V...	300	11.9	0.0880	2.2596	0 0622
" VI...	350	11.8	0 0846	2 3481	0.0598
" VII...	400	11.8	0.0856	2 5380	0.0605
" VIII...	450	10.5	0 0854	2.6860	0.0604
" IX...	500	12 0	0 0716	2 3731	0.0506

however, a second calculation was made, reducing the crushing-points from grams to kilos and tenths of kilos, which was found to materially diminish the labor without impairing the value of the results. This method was therefore followed in the case of No. 1112, in which the data were reduced to kilos and tenths for purposes of computation.

It then became a question as to how many kernels it would be necessary to crush in order to arrive at the best expression of the mean crushing-point of a given pure strain, and thereby

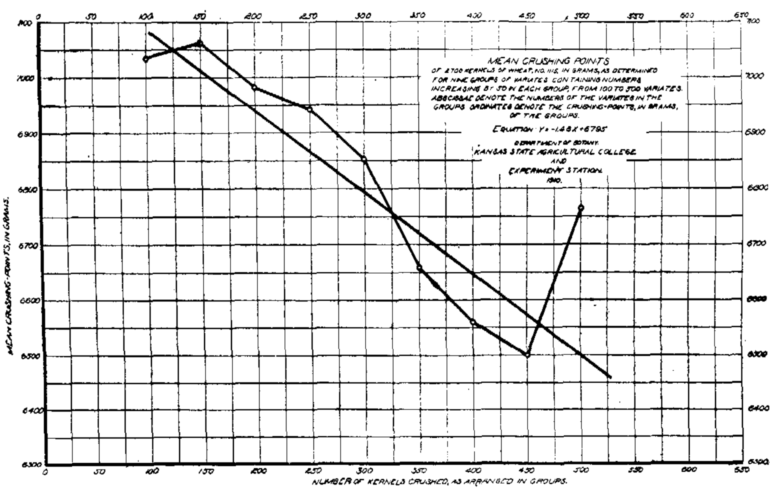


PLATE II. MEAN CRUSHING-POINTS

Of 2700 kernels of wheat No. 1115, in grams, as determined for nine groups of variates containing numbers increasing by 50 in each group, from 100 to 500 variates. Abscissæ denote the numbers of variates in the groups. Ordinates denote the crushing-points, in grams, of the groups. Equation: $y = -1.48x + 6795$.

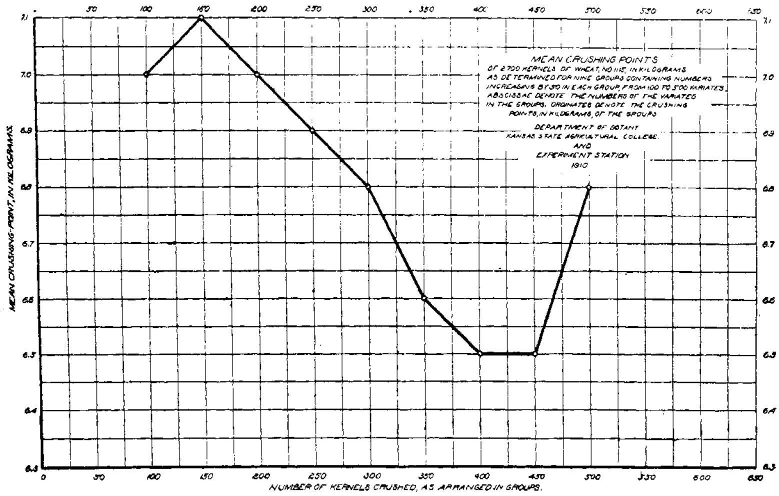


PLATE III. MEAN CRUSHING-POINTS

Of 2700 kernels of wheat No. 1115, in kilograms, as determined for nine groups containing numbers increasing by 50 in each group, from 100 to 500 variates. Abscissæ denote the numbers of the variates in the groups. Ordinates denote the crushing-points, in kilograms, of the groups.

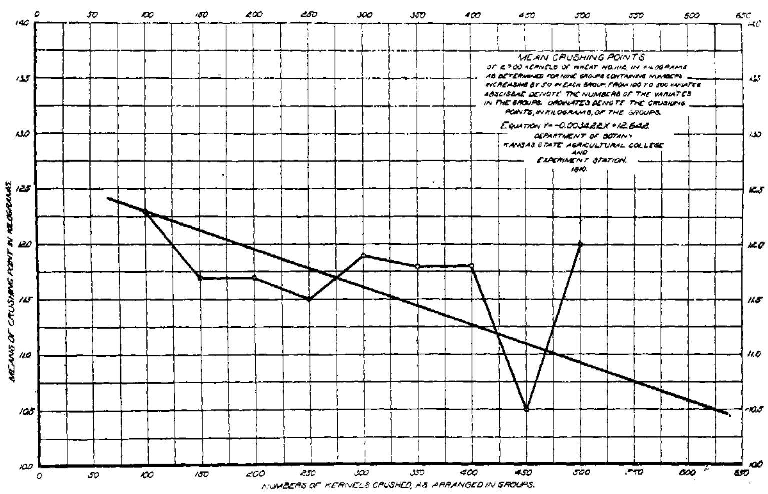


PLATE IV. MEAN CRUSHING-POINTS

Of 2700 kernels of wheat No. 1112, in kilograms, as determined for nine groups containing numbers increasing by 50 in each group, from 10) to 500 variates. Abscissæ denote the numbers of the variates in the groups. Ordinates denote the crushing-points, in kilograms, of the groups. Equation : y = $-0.003422 \times +12.642$.

of its hardness for grading purposes. By taking the means of increasingly large groups of variates as described above, and by calculating the error of the mean for each group, it was anticipated that the decrease in the error of the mean, with the increasing numbers of the variates used, would be the best possible criterion for the least number of variates necessary to be used in determining the mean crushing-point of the race.

Taking the two pure strains—No. 1115, a soft wheat, and No. 1112, a hard or semi-hard wheat—2700 kernels were crushed in the manner previously described. Plotting the mean crushing-points, we have the following curves. Plate II shows the mean crushing-points of No. 1115 plotted in grams, as the data were originally obtained. Plate III shows the same data arranged and plotted in kilos. Plate IV shows the mean crushing-points for No. 1112, which, although originally determined in grams, were plotted in kilos.

The following table gives the means of the nine groups of both 1115 and 1112, as actually determined in grams, and also as reduced to kilos, for purposes of comparison.

TABLE III.

Groups of variates.	No. of variates in groups....	No. 1115. Mean crushing-points in grams.	No. 1115. Mean crushing-points reduced to kilos and tenths of kilos.	No. 1112. Mean crushing-points in grams.	No. 1112. Mean crushing-points reduced to kilos and tenths of kilos.
Group I	100	7,035	7.00	12,277	12.3
" II	150	7,062	7.10	11,706	11.7
" III	200	6,981	7 00	11,738	11.7
" IV	250	6,943	6.90	11,496	11.5
" V	300	6,852	6.80	11,925	11 9
" VI	350	6,658	6 60	11,772	11 8
" VII	400	6,560	6.50	11,808	11.8
" VIII	450	6,499	6.50	10,496	10 5
" IX	500	6,764	6.70	12,000	12 0

No. 1115 is a derivative of a selection made in 1906 of a single head from a plot of our No. 611, called "Turkey," and received originally from the United States Department of Agriculture (U. S. Dept. No. 1676). It is a bearded wheat with white glumes, of the general "Turkey" type. No. 1112 is a derivative of a selection also made in 1906 of a single head from a plot of our No. 609, also called "Turkey," and also received originally from the United States Department of Agriculture (U. S. Dept. No. 1658). It is a wheat of the bearded type, with red glumes.

It will be noted from plate II that the mean crushing-point regularly falls from 7000 to 6500 grams in the progression from the group containing 100 to that containing 450 variates. However, the group 500 shows an astonishing rise to a mean of 6764 grams. This deflection from the straight line curve to which the remaining moments readily plot will be discussed later.

Plate III, in which the same data exhibited in plate II are plotted in kilos and round tenths, instead of in grams, reduces the irregularities in the central portion of the curve.

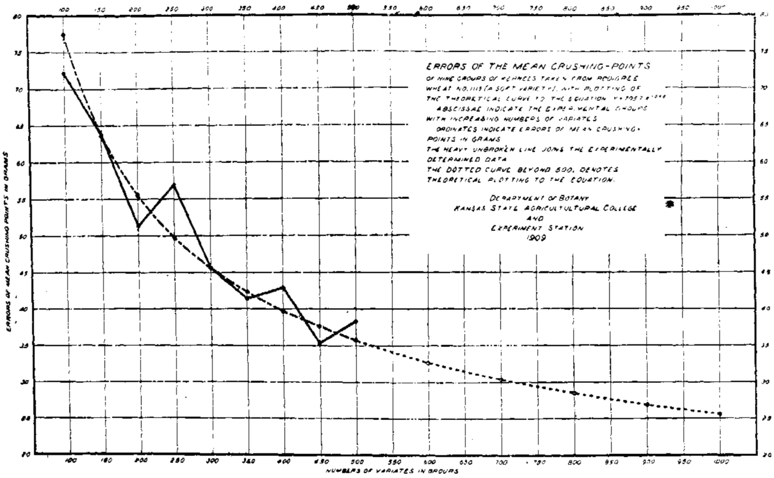


PLATE V. ERRORS OF THE MEAN CRUSHING-POINTS

Of nine groups of kernels taken from pedigree wheat No. 1115 (a soft variety), with plotting of the theoretical curve to the equation y = 705.7x −0.48.

Abscissæ indicate the experimental groups with increasing numbers of variates.

Ordinates indicate the errors of mean crushing-points, in grams.

The heavy unbroken line joins the experimentally determined data.

The dotted curve beyond 500 denotes the theoretical plotting to the equation.

In plate IV the ordinates, for convenience in plotting, are on a scale of 1 : 0.5 kilos, while in plates II and III the ordinates are plotted to a scale of 1:0.1 kilos. Plotted to the same scale as plates II and II, the irregularities in the curve of No. 1112 as compared with No. 1115 would have been very much more pronounced. It will suffice here to call attention to the same marked rise in the mean crushing-point of the 500 group of No. 1112 (in comparison with the 450 group) as was seen in No. 1115. Plotting the errors of the mean crushing-points exhibited in table II, we have for No. 1115 the curve as plotted in plate V, in which the abscissæ denote the groups, with the numbers of the variates in each, and the ordinates denote the errors of the mean crushing-point in grams. In this curve the heavy unbroken line joins the moments of the curve as determined from the experimental data, and as shown in table I under No. 1115, in the column entitled "Errors of the mean crushing-point (grams)." The broken line approximating a hyperbolic curve forms the probable best expression of the relation of the data in question, and plotted to the equation $y = 705.7x^{-0.48}$. The dotted line continues the theoretical curve beyond the last experimental datum to a datum with 1000 variates.

Likewise in plate VI is shown a plotting of the errors of the mean of wheat No. 1112, for a similar series of nine groups, having the same numbers of variates in each, as in the case of No. 1115. Here the approximate hyperbolic curve plots to the equation $122.5x^{-0.45}$.

(Note here the similarity in the exponents of the two curves, differing by only .03, which would suggest that, allowing for experimental error, the probable exponent for both would be 0.50.)

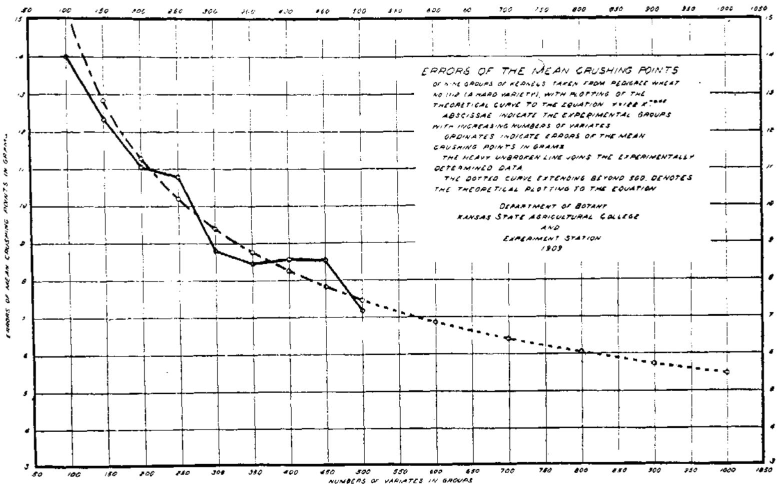


PLATE VI. ERRORS OF THE MEAN CRUSHING-POINTS

Of nine groups of kernels taken from pedigree wheat No. 1112 (a hard variety), with plotting of the theoretical curve to the equation, $y = 122x^{-0.45}$.

Abscissæ indicate the experimental groups with increasing numbers of variates.

Ordinates indicate errors of the mean crushing-points, in grams.

The heavy unbroken line joins the experimentally determined data.

The dotted curve extending beyond 500, denotes the theoretical plotting to the equation.

For those who may be interested in noting the derivation of the equation in the two cases, the mathematical data are given, as follows:

No. 1115.

Let equation be $y = bx^n$

Then, —

(1) log. y = log. b + n log. x.

Taking two points on the experimental curve, x = 150 and x = 300—

When x = 150
y = 63.71 (See table I. A.)
log. x = 2.1761
log. y = 1.8042

When x = 300
y = 45.67
log. x = 2.4771
log. y = 1.6597

Substituting in (1) and combining:

(2) 1.8042 = log. b + n (2.1761)

(3) 1.6597 = log. b + n (2.4771)
.1445 = n (−.3010)

$$n = \frac{.1445}{-.3010} = -0.48$$

Substituting in (1) and solving for b:

(5) log. b = 2.8487

(6) b = 705.7

Then, when $y = bx^n$

(7) $y = 705.7x^{-0.48}$.

Taking a point on the experimental curve, x = 100—

Substituting in (1)

(1) log. y = log. b + n log. x.

From (7), log. y = 2.8487 + (−0.48 × 2.0000)

(8) log. y = 1.8887

(9) y = 77.39, where experimental y = 72.12.

Calculating similarly for all remaining points on the experimental curve, under the assumed equation,

$y = bx^n$, or,

$y = 705.7x^{-0.48}$, we have,

TABLE IV.

	Theoretical.	Experimental.	Corrections for experimental curve + or − are additive.
Where x = 100	y = 77.39 g.	y = 72.12 g.	+ 5.27 g.
200	55.49	51.34	+ 4.15
250	49.85	56.93	− 7.08
350	42.41	41.43	+ 0.98
400	39.78	42.90	− 3.12
450	37.60	35.35	+ 2.25
500	35.74	38.37	− 2.63

Considering the close approximation of the experimental curve to the equation $y = bx^n$, when b = 705.7 and n = —0.48, and the close balancing of errors, it is concluded that this equation expresses the condition with respect to the errors of the mean crushing-points of a pure-bred wheat (No. 1115), under the conditions of the experiment.

Now, plotting the curve beyond the experimental data, we therefore have—

TABLE V.

Where x = 600	y = 32.75 g.
700	30.42
800	28.52
900	26.96
1000	25.63

Similarly in the determination of the equation for the curve of errors of mean crushing-points in wheat No. 1112:

Let equation be $y = bx^n$.

Then,

(1) log. y = log. b + n log. x.

Taking two points on a hypothetical curve, x = 275 and x = 375.

When x = 275
y = 9.78
log. x = 2.4393
log. y = 0.9903

When x = 375
y = 8.50
log. x = 2.5740
log. y = 0.9294

Substituting in (1) and combining:

(2) .9903 = log. b + n (2.4393)

(3) .9294 = log. b + n (2.5740)

.06909 = n (− 0.1347)

$$(4)\ n = \frac{.0609}{-\ .1347} = -\ 0.45$$

(5) log. b = 2.088

(6) b = 122.5

Then, when $y = bx^n$.

(7) $y = 122.5x^{-0.45}$.

Calculating all the points on the experimental curve, we have—

TABLE VI.

	Theoretical.	Experimental.	Corrections for experimental curve + or —, are additive.
Where x = 100	y = 13.98	y = 15.45	— 1 47
150	12.34	12.84	— 0.50
200	11.07	11.30	— 0.23
250	10.78	10.21	+ 0.49
300	8.80	9.40	— 0.60
350	8.46	8.77	— 0.31
400	8.56	8.26	— 0.30
450	8.54	7.83	— 0.71
500	7.16	7.48	— 0.32

Again a close fitting of the curve to the experimental data.

Plotting the theoretical curve beyond the determined data, we have—

TABLE VII.

Where x = 600	y = 17.85 g.
700	19.00
800	20.25
900	21.40
1000	22.40

It was thought desirable to determine the rate of decrease of the error of the mean crushing-point (y), for successively increasingly large numbers of kernels crushed (x); in other words, the differential of y with respect to x, or $\frac{dy}{dx}$, in the two cases under experiment, where (No. 1115) $y = 705.7x^{-0.48}$, and (No. 1112) where $y = 122.5x^{-0.48}$.

Where, $y = 705.7x^{-0.48}$, differentiating, we have

$$\frac{dy}{dx} = d\frac{(705x^{-0.48})}{dx} = -0.48\,(705.7x^{-1.48}) = \frac{-0.48 \times 705.7}{x\,1.45}, \text{ and,}$$

$$\log. \frac{dy}{dx} = \log.\ 0.48 + \log.\ 705.7 + 1.48 \text{ colog. } x\text{; and similarly,}$$

$$\text{where } y = 122.5x^{-0.45} \text{ we have } \log. \frac{dy}{dx} = \log.\ 0.45 + 1.45 \text{ colog. } x.$$

From the last two equations, determining $\frac{dy}{dx}$ for each of the groups in the two cases, Nos. 1115 and 1112, we have—

TABLE VIII. (No. 1115.)

When x = 100	$\frac{dy}{dx}$ = —.371616
150	—.203819
200	—.133048
250	—.095699
300	—.073067
350	—.058162
400	—.047732
450	—.040096
500	—.034307

TABLE IX. (No. 1112.)

When x = 100	$\frac{dy}{dx}$ = −.0691434
150	−.0384077
200	−.0253079
250	−.0183121
300	−.0140580
350	−.0112422
400	−.0092632
450	−.0078089
500	−.0067026

Having determined in terms of the differential $\frac{dy}{dx}$, in the case of No. 1115 and No. 1112, the *rate* of *change* in respect to the diminution of the error of the mean crushing-point with successively increasing numbers of grains selected for crushing, it becomes now a question as to the *relative differences* in this rate of change as the groups chosen for crushing become larger and larger, or, to use more familiar terminology, what is the *rate* of *negative acceleration* of the errors of the mean crushing-point with successively increasing values of x. We have, for No. 1115,

(1) $\frac{dy}{dx} = -0.48 \times 705.7x^{-1.48}$

(2) hence, $\frac{d^2y}{dx^2} = -1.48 \times -0.48\ (705.7x^{-2.48})$; $=501.029x^{-2.48}$

(3) log. $\frac{d^2y}{dx^2}$ = log. 501.029 − 2.48 log. x.

From which we have, in the case of No. 1115,

TABLE X.

			Differences.
When x = 100	$\frac{dy}{dx}$ = −.371616	$\frac{d^2y}{dx^2}$ = 0.00549696	
150	−.203819	0.00201102	.00348594
200	−.133048	0.00098530	.00102572
250	−.095699	0.00056654	.00041876
300	−.073067	0.00036046	.00020608
350	−.058162	0 00024594	.00011452
400	−.047732	0.00017661	.00006933
450	−.040096	0.00013187	.00004474
500	−.034307	0.00010154	.00003033

From the second differential, $\frac{d^2y}{dx^2} = 501.029x^{-0.48}$, we derive the equation for the *negative acceleration in the rate* of decrease in the error of the mean crushing-point with increased values of x. In other words, we have the equation for the *retardation in the rate of decrease* of the error of the mean, for the increasing size of the groups of variates selected for crushing.

The equation in the case of No. 1115 would be $y = \frac{501.029}{x^{-2.48}}$ or, in other words, y = the series of the second differentials as determined for table X for successive values of x.

Determining likewise the second differential in the case of No. 1112—

$$(1)\quad \frac{dy}{dx} = 0.45 \times 122.5x^{-1.45}$$

$$(2)\quad \frac{d^2y}{dx^2} = -1.45 \times 0.45 \times 122.5^{-2.45};\ = 187.739x^{-2.45}$$

$$(3)\quad \log. \frac{d^2y}{dx^2} = \log.\ 187.739 - 2.45 \log.\ x.$$

From which we have, in the case of No. 1112—

TABLE XI.

Where x =	$\frac{dy}{dx}$ =	$\frac{d^2y}{dx^2}$ =	Differences.
100	– .0691434	0.00236349	
150	– .0384077	0 00087525	.00148824
200	– .0253079	0.00043254	.00044271
250	– .0183121	0.00025038	.00018216
300	– .0140580	0.00016018	.00009020
350	– .0112422	0 00010979	.00005039
400	– .0092632	0.00007916	.00003063
450	– .0078089	0.00005931	.00001985
500	– .0067026	0 00004582	.00001349

The equation in the case of No. 1112 would be $y = \frac{187.739}{x^{-2.45}}$.

Plotting the two equations, with the numbers of variates in the groups as abscissæ, and the differentials as ordinates, we have in plates VII and VIII the graphs for the first and second differentials, respectively, for each of the strains, 1115 and 1112.

To make the matter entirely clear to those not interested in the mathematical expressions: the curve in plate VII represents the *rate* at which the *error of the mean crushing-point decreases* as we proceed to take groups of kernels for crushing which contain a larger and larger number of individuals, the number in such groups being indicated on the horizontal axis (the x-axis). This is what is known as the first differential. The rapid lessening of this differential in the first few groups is indicative of the immense gain, so far as the cutting down of the error of the mean is concerned, derived from taking say, 150 or 250 kernels, instead of 100. The rapid slowing down of this rate of decrease when we reach 350 kernels is sufficiently plain. It therefore appears evident that our empirical choice of 350 kernels as the optimum practical number for crushing, to determine the mean crushing-point of a given pure race of wheat, is perfectly justifiable.

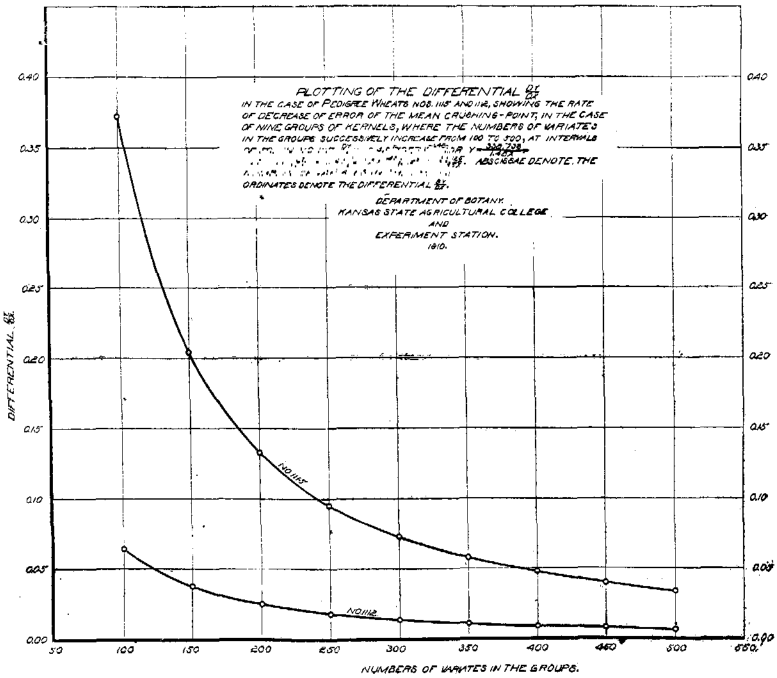


PLATE VII. PLOTTING OF THE DIFFERENTIAL $\frac{DY}{DX}$.

In the case of pedigree wheats Nos. 1115 and 1112, showing the rate of decrease of error of the mean crushing-point, in the case of nine groups of kernels, where the numbers of variates in the groups successively increase from 100 to 500, at intervals of 50.

In No. 1115, $\frac{dy}{dx} = -0.48\ (705.7x^{-1.48})$, or $y = \frac{338.736}{1.48x}$.

In No. 1112, $\frac{dy}{dx} = -0.45\ (122.5x^{-1.45})$, or $y = \frac{55.125}{1.45x}$.

Abscissæ denote the numbers of variates in the groups.

Ordinates denote the differential $\frac{dy}{dx}$.

In plate VIII we have likewise the graph or curve for the second differential, which expresses not the *rate of decrease* itself, but the *rate of speed* at which *that rate slows down.*

In more general terms, the first differential expresses *velocity,* while the second differential expresses *acceleration.*

The velocity may be an increasing or a decreasing one; and likewise, instead of *positive* acceleration, we may have *negative* acceleration or retardation.

In the present case we are dealing of course with a decreasing velocity and a negative acceleration.

The curve of the second differential shows finally the fundamental object of the investigation—to discover *at what point*

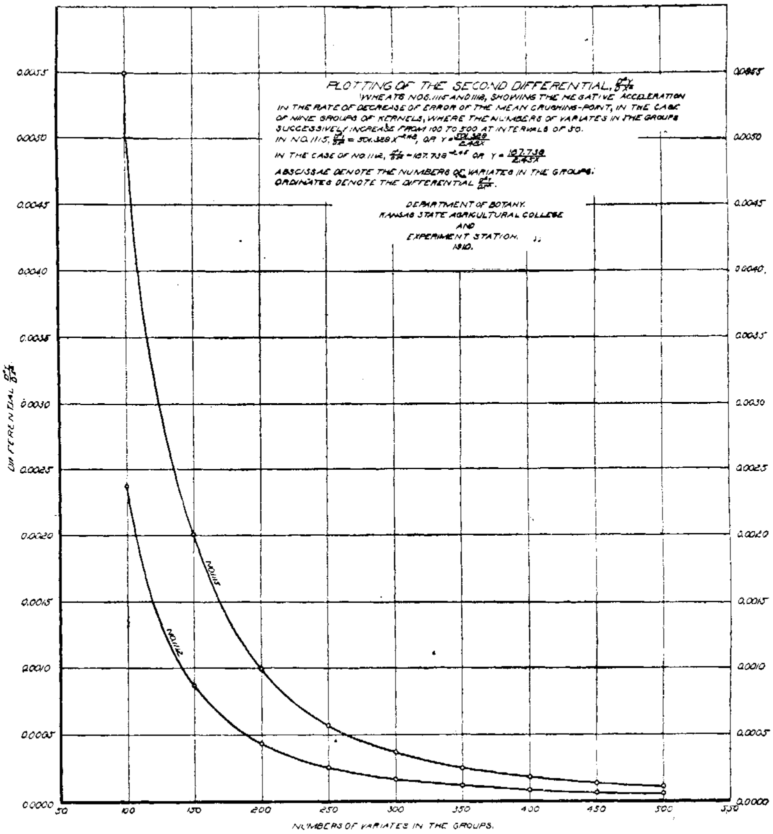


PLATE VIII. PLOTTING OF THE SECOND DIFFERENTIAL $\frac{D^2Y}{DX^2}$.

Wheats Nos. 1115 and 1112 showing the negative acceleration in the rate of decrease of error of the mean crushing-point, in the case of nine groups of kernels, where the numbers of variates in the groups successively increase from 100 to 500 at intervals of 50.

In No. 1115, $\frac{d^2y}{dx^2} = 501.329x^{-2.45}$, or $y = \frac{501.329}{2.48x}$.

In the case of No. 1112, $\frac{d^2y}{dx^2} = 187.739^{-2.45}$ or $y = \frac{187.739}{2.45x}$.

Abscissæ denote the number of variates in the groups.

Ordinates denote the differential $\frac{d^2y}{dx^2}$.

the rate at which the diminishing error falls off—becomes itself so slight as to be negligible.

The graph in plate VIII shows most convincingly that the critically important point in this respect is more than reached at the 350-variate group.

Referring back to plate II and table I, it will be noted that the mean crushing-point, which falls off steadily to 6499 grams at the 450-variate point, suddenly rises at the 500-variate point to 6764. This sudden deflection of the curve calls for investigation, as it clearly does not harmonize with the other points of the curve. In order to analyze the data more precisely, a mean was taken of each successive hundred variates in the original raw data of individual crushing-points. These means of twenty-seven successive hundreds are given as follows:

TABLE XII.

Group.		Mean crushing-point.
First	100	7025 grams.
Second	100	7112 "
Third	100	7043 "
Fourth	100	6977 "
Fifth	100	6938 "
Sixth	100	7056 "
Seventh	100	6784 "
Eighth	100	6885 "
Ninth	100	6962 "
Tenth	100	6708 "
Eleventh	100	6726 "
Twelfth	100	6681 "
Thirteenth	100	6461 "
Fourteenth	100	6422 "
Fifteenth	100	6667 "
Sixteenth	100	6773 '
Seventeenth	100	6451 "
Eighteenth	100	6605 "
Nineteenth	100	6447 "
Twentieth	100	6434 "
Twenty-first	100	6099 "
Twenty-second	100	6865 "
Twenty-third	100	6731 "
Twenty-fourth	100	6673 "
Twenty-fifth	100	6598 "
Twenty-sixth	100	6639 "
Twenty-seventh	100	6983 "

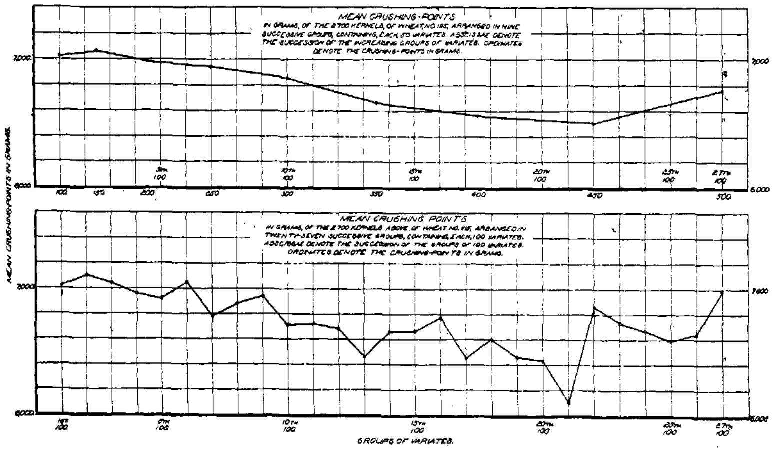


PLATE IX. (Upper graph.) MEAN CRUSHING-POINTS,

In grams, of the 2700 kernels of wheat No. 1115, arranged in nine successive groups, containing each 50 variates more than the next preceding group. Abscissæ denote the succession of the increasing groups of variates. Ordinates denote the crushing-points, in grams.

PLATE IX. (Lower graph.) MEAN CRUSHING-POINTS,

In grams, of the 2700 kernels above, of wheat No. 1115, arranged in twenty-seven successive groups, containing each 100 variates and showing the individual variation of the groups. Abscissæ denote the succession of the groups of 100 variates. Ordinates denote the crushing-points, in grams.

Plotting these, we have the lower graph shown on plate IX, with the graph for the mean crushing-points of the nine successively increasing groups plotted on the same scale above for comparison. The lower curve shows the individual variation in the groups of 100 each, and it is to be seen that whereas the eleven successive hundreds, from the tenth to the twentieth, inclusive, all had means beween 6400 and 6800 grams, the last five successive hundreds, from the twenty-second to the twenty-seventh, inclusive, had means lying between 6600 and 7000. The upper curve thus having its components analyzed in the lower one, renders it possible to see exhibited there the details of the sudden rise in the mean crushing-point of the last group previously referred to, in the upper curve of plate IX. The abrupt rise of this 500 group, as contrasted with its immediately preceding one containing 450 variates, is especially striking. No way has been found to account for this sudden upward displacement of the mean crushing-points in the last five groups of 100 variates each, unless on the theory that the prolonged delay of the kernels of the last series in the desiccator while awaiting crushing would tend to increase the relative hardness of this particular lot. However, the sudden increase in mean crushing-point in this series, as shown in the upper graph of plate IX, and more strikingly on the larger scale of plate II, does not affect the general condition of the curve, and it may therefore, as the one erratic moment, be neglected.

Finally, the investigation has in mind as its main ultimate objects:

First. The discovery of the correlation of hardness, as determined in terms of the mean crushing-point, with certain chemical factors of economic importance, to be discussed in a succeeding bulletin.

Second. It is desired to assist in laying the foundation for a standard system of grading wheat on the basis of economic values to be determined quantitatively, That there is a definite relation between hardness in wheat and these economic factors is generally admitted.

The present machine used in determining the crushing-point of wheat, while perfectly accurate, needs to be modified in such a way as to increase the speed of its operation, and a number of changes are being considered to this end. Any scientific method of wheat grading must admit of great rapidity of operation, in order to be of practical commercial value.